hw set 26
sect 8.7
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9. $f(x) = 1 + x + x^2$, $a = 2$

$f'(x) = 1 + 2x$

$f''(x) = 2$

$f'''(x) = 0$

$$f(x) = f(a) + f'(a)(x-2) + \frac{f''(a)}{2}(x-2)^2$$
$$= 7 + 5(x-2) + (x-2)^2$$

10. $f(x) = x^3$, $a = -1$

$f'(x) = 3x^2$

$f''(x) = 6x$

$f'''(x) = 6$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-$$
$$= -1 + 3(x+1) - 3(x+1)^2 + (x+1)^3$$

11. $f(x) = e^x$, $a = 3$

$f^{(n)}(x) = e^x$

$f^{(n)}(3) = e^3$

$$f(x) = e^3 + e^3(x-3) + \frac{e^3}{2}(x-3)^2 + \frac{e^3}{3!}(x-3)^3 + \frac{e^4}{\cancel{4}!}(x-3)^4 + \dots$$
$$= \sum_{n=0}^{\infty} \frac{e^3}{n!}(x-3)^n$$

12. $f(x) = \ln x$, $a = 2$

$f'(x) = 1/\cancel{x}$ $\qquad f^{(4)}(x) = -6/x^4$

$f''(x) = -1/x^2$ $\qquad \vdots$

$f'''(x) = 2/x^3$ $\qquad f^{(n)}(x) = \dfrac{(-1)^{n-1} n!}{x^n}$

$$f(x) = \ln(2) + \tfrac{1}{2}(x-2) - \tfrac{1}{8}(x-2)^2 + \dots + \frac{(-1)^{n-1} n!}{x^n}(x-2)^n + \dots$$

13. $f(x) = \cos x$, $a = \pi$

$$f(x) = \cos(\pi) + \frac{(-\sin(\pi))}{1}(x-\pi) + \frac{-\cos(\pi)}{2}(x-\pi)^2 + \dots$$
$$= -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{4!} + \frac{(x-\pi)^6}{6!} + \dots$$

22. $f(x) = \sin(x^4)$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin(x^4) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^4)^{2n+1}}{(2n+1)!}$$
$$= \sum_{n=0}^{\infty} (-1) \frac{x^{8n+4}}{(2n+1)!}$$

23. $f(x) = x^2 e^{-x}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$f(x) = x^2\left(1 - \frac{x}{1} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots\right)$$
$$= x^2 - x^3 + \frac{x^4}{2!} - \frac{x^5}{3!} + \frac{x^6}{4!} + \dots$$
$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{n!}$$

27. cancel

34. $\int \frac{\sin x}{x}\,dx$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\int \frac{\sin x}{x}\,dx = \int 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$= \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{2n}$$

$$= \sum_{n=0} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{2n+1}}{2n+1}$$

$$= x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \dots$$

41. $\lim\limits_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots = \sum (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\lim_{x \to 0} \frac{x - \left(x - x^3/3 + x^5/5 - x^7/7 + \dots\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\left(x^3/3 - x^5/5 + x^7/7 - \dots\right)}{x^3}$$

$$= \lim_{x \to 0} \left(\frac{1}{3} - \frac{x^2}{5} + \frac{x^4}{7} - \frac{x^6}{9} + \dots\right)$$

$$= 1/3$$